A Magnetodynamic Error Criterion and an Adaptive Meshing Strategy for Eddy Current Evaluation

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In order to address their considerable impacts on both the energy efficiency and performance requirements, eddy current modeling and its accuracy are discussed from a thermodynamic approach. Coupled with an adaptative meshing strategy, some numerical results are given on an induction machine.

Index Terms **— Eddy currents, Finite element analysis, Adaptive mesh refinement, Induction motors.**

I. Introduction

EDDY CURRENTS are at the origin of losses and signal distorsions in power electrical devices from deep within the conducting materials to power electrical device scale. At the design level, reducing losses requires therefore relevant behavior laws and efficient numerical techniques, including inspection of the solution. While adaptive meshing strategies have been extensively used in static cases [1,2], they remain little explored in transient [3]. In the following, an energy-based error criterion well-fitted for eddy current modeling is obtained from a thermodynamic derivation of electromagnetism. Associated with an adaptive meshing strategy, some valuable results are carried out in an induction machine case-study.

II. Variational Formulation

Denoting, as a general rule in this paper, variational parameters or functional thanks to *italic* fonts whereas roman ones specify their value at the minimum, the magnetodynamic behavior of any electrical system is derived from the functional, expressing the difference between the mechanical power received by the field from the actuators $\mathrm{P}_{\mathrm{mech}}$ and the variations with time of the Gibbs' free energy G [4]:

$$\mathrm{P}_{\mathrm{mech}}-\frac{\mathrm{dG}}{\mathrm{d}t}=\min_{\boldsymbol{H},\boldsymbol{E}}\left(\int_C \sigma^{-1}(\mathbf{curl}\boldsymbol{H})^2\,\mathrm{d}^3x+\frac{\mathrm{d}}{\mathrm{d}t}\int\left(\mathbf{B}\cdot\boldsymbol{H}+\mathbf{D}\cdot\boldsymbol{E}\right)\mathrm{d}^3x\right) \quad (1)$$

where the functional in the RHS exhibits:

- the magnetic field $\boldsymbol{H}$ related to free and displacement currents according to the Maxwell-Ampere equation. The quasi-static approximation enforce $\mathbf{D}\equiv 0$ in conductors;
- the Joule losses P_{J} in conductors. This term is even to respect losses with time inversion (σ^{-1} is the resistivity);
- the variation with time of the electromagnetic energy coupling the field with the generator I and the mass V_0;
- the magnetic $\mathbf{B}(\boldsymbol{h})$ and electrostatic $\mathbf{D}(\boldsymbol{e})$ behavior laws derived from thermostatic equilibrium of the Gibbs potential:

$$G(\mathrm{T},\mathrm{I},\mathrm{V}_0)=G_m+G_e=\int\left(\int_0^{\boldsymbol{H}}(-\boldsymbol{B})\cdot\delta\boldsymbol{h}+\int_0^{\boldsymbol{E}}(-\boldsymbol{D})\cdot\delta\boldsymbol{e}\right)\mathrm{d}^3x \quad (2)$$

Extending the electric field in the conductor according to Ohm's law: $\mathbf{E}=\sigma^{-1}\mathbf{J}-\mathbf{V}\times\mathbf{B}$, Faraday's law: $\mathbf{curl}\ \mathbf{E}=-\partial_t\mathbf{B}$ may be viewed as acting locally to check globally the optimal tendency towards reversibility expressed by (1). Hence, the functional (1) balances the variations with time of the co-energy (–G) and the mechanical power supplied to the field $\mathrm{P}_{\mathrm{mech}}$. In order to consider sub-systems for design purpose, it is convenient to introduce the electrical power of the domain Ω:

$$P_{\mathrm{elec}}(\Omega)=-\oint_{\partial\Omega}(\boldsymbol{E}\times\boldsymbol{H})\cdot\mathbf{n}\,\mathrm{d}^2x \quad (3)$$

After some calculations, it follows:

$$\begin{aligned}P_{\mathrm{elec}}(\Omega)=&-\int_\Omega(\mathbf{curl}\boldsymbol{E}+\partial_t\boldsymbol{B})\cdot\boldsymbol{H}\,\mathrm{d}^3x+\int_\Omega(\mathbf{curl}\boldsymbol{H}-\boldsymbol{J}-\partial_t\boldsymbol{D})\cdot\boldsymbol{E}\,\mathrm{d}^3x\\&+\int_{C\subset\Omega}\boldsymbol{J}\cdot\left(\boldsymbol{E}-\sigma^{-1}\boldsymbol{J}+\mathbf{V}\times\boldsymbol{B}\right)\mathrm{d}^3x+P_{\mathrm{J}}(\Omega)+\frac{\mathrm{d}F}{\mathrm{d}t}(\Omega)+\int_{C\subset\Omega}\mathbf{V}\cdot(\boldsymbol{J}\times\boldsymbol{B})\mathrm{d}^3x\\&-\sum_i\oint_{\partial\Omega_i}\left([\boldsymbol{E}\times\boldsymbol{H}]\cdot\mathbf{n}-\left[\int_0^{\boldsymbol{B}}\boldsymbol{H}\cdot\delta\mathbf{b}+\int_0^{\boldsymbol{D}}\boldsymbol{E}\cdot\delta\mathbf{d}\right](\mathbf{V}_i\cdot\mathbf{n})\right)\mathrm{d}^2x\end{aligned} \quad (4)$$

where F is the Helmoltz' potential and the brackets $[\cdot]$ denote the discontinuities occurring at the interfaces $\partial\Omega_i\subset\Omega$. At the minimum of the functional (1), the Maxwell equation set and Ohm's law are checked so that:

- the first three residual terms vanish in (4). After some tedious calculations on the motion induced-interface discontinuities, the last two terms provide the mechanical power in a form close to the Maxwell stress tensor:

$$\begin{aligned}-\mathrm{P}_{\mathrm{mech}}(\Omega)=&\sum_i\oint_{\partial\Omega_i}(\mathbf{n}\cdot\mathbf{V})[\mathbf{D}\times\mathbf{B}]\cdot\mathbf{V}_i\,\mathrm{d}^2x+\sum_i\int_{C_i}(\mathbf{J}\times\mathbf{B})\cdot\mathbf{V}_i\,\mathrm{d}^3x\\&+\sum_i\oint_{\partial\Omega_i}\left([(\mathbf{B}\cdot\mathbf{n})\mathbf{H}+(\mathbf{D}\cdot\mathbf{n})\mathbf{E}]-\left[\int_0^{\mathbf{H}}\mathbf{B}\cdot\delta\mathbf{h}+\int_0^{\mathbf{E}}\mathbf{D}\cdot\delta\mathbf{e}\right]\cdot\mathbf{n}\right)\cdot\mathbf{V}_i\,\mathrm{d}^2x\end{aligned} \quad (5)$$

- the contribution of Ω to (1) reads:

$$\mathrm{P}_{\mathrm{mech}}(\Omega)+\mathrm{P}_{\mathrm{elec}}(\Omega)-\frac{\mathrm{dG}}{\mathrm{d}t}(\Omega) \quad (6)$$

The Finite Element Method consists in building an approximation of (1) and (2) but with a finite number of degrees of freedom chosen on a mesh. Whereas the stationary conditions expressed on (1) and (2) provide an approximation of the fields, the consistency of the solution with energy conservation may be assessed through the local deviation of the Poynting's equation:

$$\varepsilon(\Omega) = P_{\text{elec}}(\Omega) - P_{\text{J}}(\Omega) - \frac{\mathrm{d}F}{\mathrm{d}t}(\Omega) + \mathrm{P}_{\text{mech}}(\Omega) \tag{7}$$

Strictly enforcing two relations among Maxwell-Ampere or Maxwell-Faraday equations and Ohm's law, the error criterion (7) highlights the elements where the third one is ill-checked. In the following, an iterative remeshing technique [2] is coupled with the criterion (7) and applied to an induction machine (Fig. 1). A selection of the worst elements in the sense of the criterion is refined at each iteration. For each targeted element, a node is added, the element is split and neighbor elements are also split to ensure conformity of the mesh. A mesh optimization may be performed at the end of the iteration to improve the aspect ratio of the mesh. The performance of such process is tested to find the best compromise between accuracy and computation time.

III. Numerical results

For time-harmonic magnetodynamic linear problems, a phasor-complex representation of the field is adopted, *i.e.* $\mathrm{d}/\mathrm{d}t \to \mathrm{j}\omega$. Denoting with $\overline{}$ the complex representation, the electrical power and the Helmoltz' free energy in conductors read respectively:

$$\overline{P}_{\text{elec}}(\Omega) = -\oint_{\partial\Omega} \left(\overline{\boldsymbol{E}} \times \overline{\boldsymbol{H}}^{*}\right) \cdot \mathbf{n}\, \mathrm{d}^2 x \tag{8}$$

$$\overline{F}(\Omega) = \int_{\Omega} \frac{1}{2} \mu \overline{\boldsymbol{H}} \cdot \overline{\boldsymbol{H}}^{*}\, \mathrm{d}^3 x \tag{9}$$

so that the criterion (7) is as follow adapted:

$$\varepsilon(\Omega) = \left|\mathrm{Re}\left(\overline{P}_{\text{elec}}(\Omega)\right) - P_{\text{J}}(\Omega) + \mathrm{P}_{\text{mech}}(\Omega) + \mathrm{j}\left(\mathrm{Im}\left(\overline{P}_{\text{elec}}(\Omega)\right) - 2\omega\overline{F}(\Omega)\right)\right| \tag{10}$$

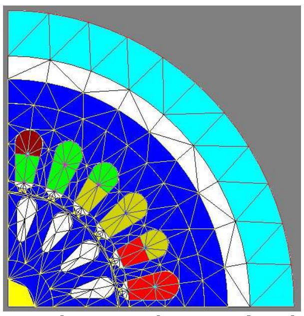

Fig. 1. Induction machine: initial mesh.

The adaptive solving procedure is applied to the evaluation of the torque of an induction machine (4 poles, 24 slots, rated speed 1410 rpm at 0.6 slip) with a coarse mesh as a starting point as shown in Fig. 1. With a very fine mesh solution as a reference, the accuracy of the adaptive refinement is checked looking at the average deviation on the torque vs. slip curve (Fig. 2). Table I shows that the accuracy on the torque is improved as the mesh is refined with the iterations. With Fig. 3, it also highlights the impact of the mesh optimization process. When using optimization, each iteration requires more computation time because it adds more nodes and because of the optimization process itself. This is penalizing to get 3% accuracy; this is very efficient to get high accuracy.

TABLE I

Number of iterations and computation time to reach a given accuracy on torque value at slip value of 0.6

Torque accuracy	Non-optimized mesh	Optimized mesh
<3%	5 iterations 25s	2 iterations 35s
<1%	20 iterations 170s	2 iterations 35s

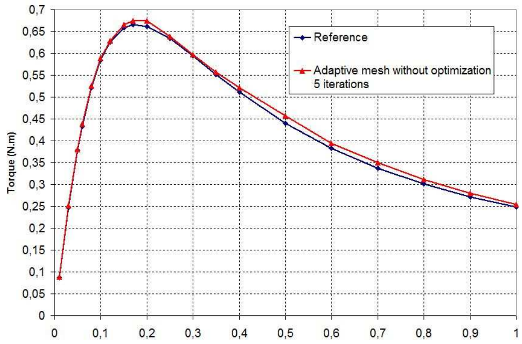

Fig. 2. Torque *vs.* slip characteristic using adaptive mesh without optimization Notice the convergence of the curve after 5 iterations for any slip value.

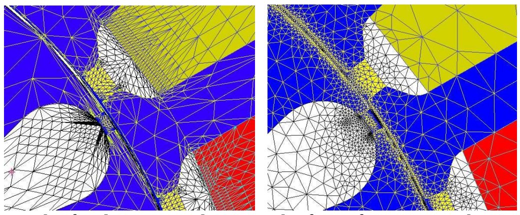

Fig. 3. Mesh of Induction machine: mesh after refinement: without mesh optimization (left) and with optimization (right)

IV. Conclusion

The developed criterion and mesh refinement strategy allows to correctly evaluate the eddy currents in the rotor bars of an induction machine and the resulting magnetic torque. At this stage, we have chosen to check the accuracy through a suggestive value like the torque; in the full paper, the convergence will also assessed considering the evolution of (6); other examples will also be shown. As a perspective especially dedicated to open-boundary problems, further calculations of the electric field within the dielectric region can be developed, to derive fully energy-based adaptive meshing strategies within FEM.

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